2016 AMC 10B

Prepared by SEM AMC Club

Problem 1

What is the value of $\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\frac{1}{2}$?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 2\qquad\textbf{(C)}\ \frac{5}{2}\qquad\textbf{(D)}\ 10\qquad\textbf{(E)}\ 20$

Problem 2

If $n\heartsuit m=n^3m^2$, what is $\frac{2\heartsuit 4}{4\heartsuit 2}$?

$\textbf{(A)}\ \frac{1}{4}\qquad\textbf{(B)}\ \frac{1}{2}\qquad\textbf{(C)}\ 1\qquad\textbf{(D)}\ 2\qquad\textbf{(E)}\ 4$

Problem 3

Let $x=-2016$. What is the value of $\Bigg\vert\Big\vert |x|-x\Big\vert-|x|\Bigg\vert-x$?

$\textbf{(A)}\ -2016\qquad\textbf{(B)}\ 0\qquad\textbf{(C)}\ 2016\qquad\textbf{(D)}\ 4032\qquad\textbf{(E)}\ 6048$

Problem 4

Zoey read 15 books, one at a time. The first book took her 1 day to read, the second book took her 2 days to read, the third book took her 3 days to read, and so on, with each book taking her 1 more day to read than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day of the week did she finish her 15th book?

$\textbf{(A)}\ \text{Sunday}\qquad\textbf{(B)}\ \text{Monday}\qquad\textbf{(C)}\ \text{Wednesday}\qquad\textbf{(D)}\ \text{Friday}\qquad\textbf{(E)}\ \text{Saturday}$

Problem 5

The mean age of Amanda's 4 cousins is 8, and their median age is 5. What is the sum of the ages of Amanda's youngest and oldest cousins?

$\textbf{(A)}\ 13\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 19\qquad\textbf{(D)}\ 22\qquad\textbf{(E)}\ 25$

Problem 6

Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S?

$\textbf{(A)}\ 1\qquad\textbf{(B)}\ 4\qquad\textbf{(C)}\ 5\qquad\textbf{(D)}\ 15\qquad\textbf{(E)}\ 20$

Problem 7

The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?

$\textbf{(A)}\ 75\qquad\textbf{(B)}\ 90\qquad\textbf{(C)}\ 135\qquad\textbf{(D)}\ 150\qquad\textbf{(E)}\ 270$

Problem 8

What is the tens digit of $2015^{2016} - 2017$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 8$

Problem 9

All three vertices of $\triangle ABC$ are lying on the parabola defined by $y = x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 16$

Problem 10

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

$\textbf{(A)}\ 14.0 \qquad \textbf{(B)}\ 16.0 \qquad \textbf{(C)}\ 20.0 \qquad \textbf{(D)}\ 33.3 \qquad \textbf{(E)}\ 55.6$

Problem 11

Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

$\textbf{(A)}\ 256 \qquad \textbf{(B)}\ 336 \qquad \textbf{(C)}\ 384 \qquad \textbf{(D)}\ 448 \qquad \textbf{(E)}\ 512$

Problem 12

Two different numbers are selected at random from $(1, 2, 3, 4, 5)$ and multiplied together. What is the probability that the product is even?

$\textbf{(A)}\ 0.2 \qquad \textbf{(B)}\ 0.4 \qquad \textbf{(C)}\ 0.5 \qquad \textbf{(D)}\ 0.7 \qquad \textbf{(E)}\ 0.8$

Problem 13

At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 100 \qquad \textbf{(E)}\ 160$

Problem 14

How many squares whose sides are parallel to the axis and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y = \pi x$, the line $y = -0.1$ and the line $x = 5.1$?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 41 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 50 \qquad \textbf{(E)}\ 57$

Problem 15

All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 16

The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

$\textbf{(A)}\ \frac{1+\sqrt{5}}{2} \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ \sqrt{5} \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 17

All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

$\textbf{(A)}\ 312 \qquad \textbf{(B)}\ 343 \qquad \textbf{(C)}\ 625 \qquad \textbf{(D)}\ 729 \qquad \textbf{(E)}\ 1680$

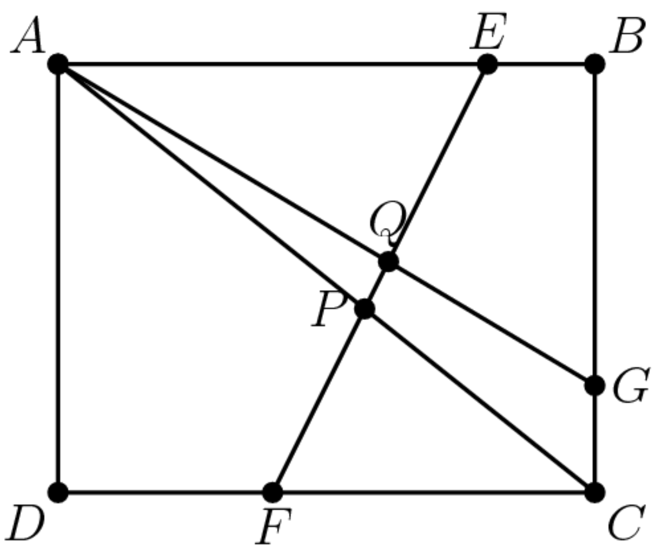
Problem 18

In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 7$

Problem 19

Rectangle $ABCD$ has $AB = 5$ and $BC = 4$. Point E lies on $\overline{AB}$ so that $EB = 1$, point G lies on $\overline{BC}$ so that $CG = 1$. and point F lies on $\overline{CD}$ so that $DF = 2$. Segments $\overline{AG}$ and $\overline{AC}$ intersect $\overline{EF}$ at Q and P, respectively. What is the value of $\frac{PQ}{EF}$?



$\textbf{(A)}\ \frac{\sqrt{13}}{16} \qquad \textbf{(B)}\ \frac{\sqrt{2}}{13} \qquad \textbf{(C)}\ \frac{9}{82} \qquad \textbf{(D)}\ \frac{10}{91} \qquad \textbf{(E)}\ \frac{1}{9}$

Problem 20

A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2,2)$ to the circle of radius 3 centered at $A'(5,6)$. What distance does the origin $O(0,0)$, move under this transformation?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ \sqrt{13} \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$

Problem 21

What is the area of the region enclosed by the graph of the equation $x^2+y^2=|x|+|y|?$

$\textbf{(A)}\ \pi+\sqrt{2} \qquad \textbf{(B)}\ \pi+2 \qquad \textbf{(C)}\ \pi+2\sqrt{2} \qquad \textbf{(D)}\ 2\pi+\sqrt{2} \qquad \textbf{(E)}\ 2\pi+2\sqrt{2}$

Problem 22

A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

$\textbf{(A)}\ 385 \qquad \textbf{(B)}\ 665 \qquad \textbf{(C)}\ 945 \qquad \textbf{(D)}\ 1140 \qquad \textbf{(E)}\ 1330$

Problem 23

In regular hexagon $ABCDEF$, points W, X, Y, and Z are chosen on sides $\overline{BC}$, $\overline{CD}$, $\overline{EF}$, and $\overline{FA}$ respectively, so lines AB, ZW, YX, and ED are parallel and equally spaced. What is the ratio of the area of hexagon $WCXYFZ$ to the area of hexagon $ABCDEF$?

$\textbf{(A)}\ \frac{1}{3} \qquad \textbf{(B)}\ \frac{10}{27} \qquad \textbf{(C)}\ \frac{11}{27} \qquad \textbf{(D)}\ \frac{4}{9} \qquad \textbf{(E)}\ \frac{13}{27}$

Problem 24

How many four-digit integers $abcd$, with $a \neq 0$, have the property that the three two-digit integers $ab<bc<cd$ form an increasing arithmetic sequence? One such number is 4692, where $a=4$, $b=6$, $c=9$, and $d=2$.

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 20$

Problem 25

Let $f(x) = \sum_{k=2}^{10}(\lfloor kx \rfloor -k \lfloor x \rfloor)$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to r. How many distinct values does $f(x)$ assume for $x \ge 0$?

$\textbf{(A)}\ 32 \qquad \textbf{(B)}\ 36 \qquad \textbf{(C)}\ 45 \qquad \textbf{(D)}\ 46 \qquad \textbf{(E)}\ \text{infinitely many}$