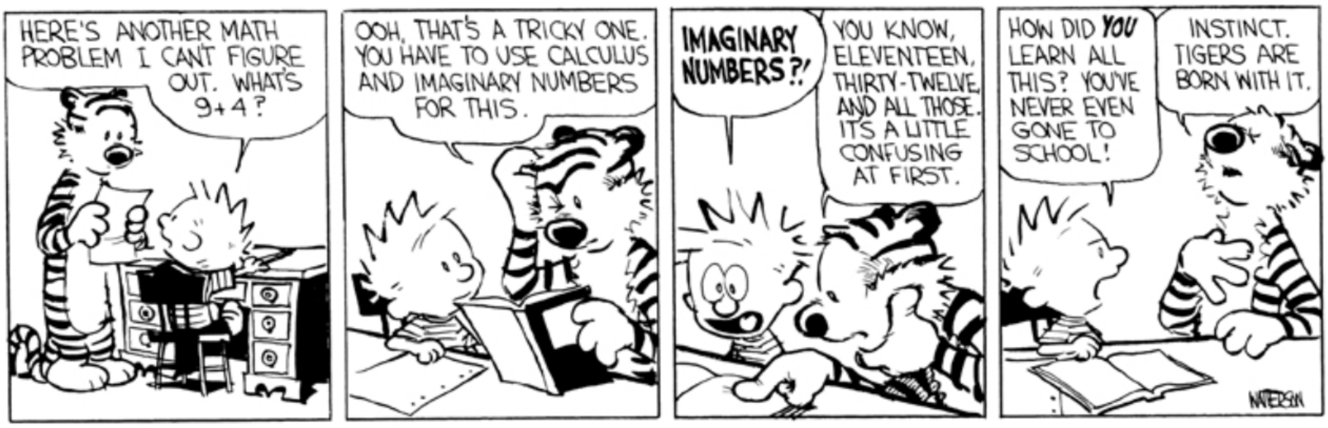
AMC Club Practice Problems



We have studied multiple topics over the past few weeks. Instead of a new lesson today, we are going to work on some problems covering all of the topics we have learned over the previous lessons. Topics covered include Vieta's Formulas, Sequences and Series, Complimentary Counting, PIE and overcounting.

1. Let r_1, r_2 and r_3 be the roots of the polynomial $5x^3 - 11x^2 + 7x + 3$. Evaluate $r_1^3 + r_2^3 + r_3^3$. *(Source: Brilliant)*

2. Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer? *(Source: AMC)*

   $(\text{A})\ \frac{1}{6}$ $\quad(\text{B})\ \frac{1}{3}$ $\quad(\text{C})\ \frac{1}{2}$ $\quad(\text{D})\ \frac{5}{8}$ $\quad(\text{E})\ \frac{3}{4}$

3. A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations? *(Source: AMC)*

   $\text{(A) } 36 \qquad \text{(B) } 40 \qquad \textbf{(C) } 44 \qquad \text{(D) } 48 \qquad \text{(E) } 52$

   Hint: notice the number of permutations where $a_1 + a_2 < a_4 + a_5$ is equal to the number of permutations where $a_1 + a_2 > a_4 + a_5$. Try using complimentary counting, and counting the permutations such that $a_1 + a_2 = a_4 + a_5$.

4. Postman Pete has a pedometer to count his steps. The pedometer records up to 99999 steps, then flips over to 00000 on the next step. Pete plans to determine his mileage for a year. On January 1 Pete sets the pedometer to 00000. During the year, the pedometer flips from 99999 to 00000 forty-four times. On December 31 the pedometer reads 50000. Pete takes 1800 steps per mile. Which of the following is closest to the number of miles Pete walked during the year? *(Source: AMC)*

   $\textbf{(A) } 2500 \qquad \textbf{(B) } 3000 \qquad \textbf{(C) } 3500 \qquad \textbf{(D) } 4000 \qquad \textbf{(E) } 4500$

5. Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7? *(Source: AMC)*

   $\text{(A) } \frac{1}{9} \qquad \text{(B) } \frac{1}{8} \qquad \text{(C) } \frac{1}{6} \qquad \text{(D) } \frac{2}{11} \qquad \text{(E) } \frac{1}{5}$

6. How many four-digit positive integers have at least one digit that is a 2 or a 3? *(Source: AMC)*

   (A) 2439 (B) 4096 (C) 4903 (D) 4904 (E) 5416

7. A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers chose spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires 2 adjacent spaces. What is the probability that she is able to park? *(Source: AMC)*

   (A) $\frac{11}{20}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{81}{140}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{17}{28}$

8. An infinite geometric series has sum 2005. A new series, obtained by squaring each term of the original series, has 10 times the sum of the original series. The common ratio of the original series is $\frac{m}{n}$ where m and n are relatively prime integers (basically, the fraction is in lowest terms). Find $m+n$. *(Source: AIME)*

9. Jenn randomly chooses a number J from $1, 2, 3, \ldots, 19, 20$. Bela then randomly chooses a number B from $1, 2, 3, \ldots, 19, 20$ distinct from J. The value of $B - J$ is at least 2 with a probability that can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$. *(Source: AIME)*

10. A convex polyhedron P has 26 vertices, 60 edges, and 36 faces, 24 of which are triangular and 12 of which are quadrilaterals. A space diagonal is a line segment connecting two non-adjacent vertices that do not belong to the same face. How many space diagonals does P have? *(Source: AIME)*

Answer Key:

1. $-\frac{49}{125}$
2. D
3. D
4. A
5. D
6. E
7. E
8. 802
9. 029
10. 241